OHMIC HEATING OF A DENSE HYDROGEN PLASMA

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We investigate the possibility of ohmic heating of a hydrogen plasma in a quasi-stationary discharge without an external magnetic field (discharge time 180 μsec, I_{max} = 160 kA). We obtain a stable column, free from impurities, with a temperature of about 10 eV and a density of $2 \cdot 10^{18}$ cm^{-3}. A numerical calculation of the heating process is compared with experimental data.

In view of the various applications of a high-temperature and fairly dense plasma, systems with Joule heating are of interest. In the early stages of research on controlled thermonuclear synthesis the possibility of using a quasi-stationary discharge in straight and toroidal tubes for this purpose was investigated. However, interest in such a method of heating was discouraged by the reduction in the heating efficiency with temperature increase. In the ideal case—with losses due only to bremsstrahlung and in the absence of a longitudinal magnetic field—the current is limited to $\sim 10^6$ A, which corresponds to a temperature of ~100 eV. In addition, in experiments on slow z-pinches the plasma filament at an early stage of heating became subject to numerous magnetohydrodynamic instabilities.

The gas-insulation system proposed by Alfven [1] is an attempt to obtain a stable plasma column in the simplest z-pinch system without a longitudinal magnetic field. The role of the cold sheath in this case, however, reduces largely to restriction of access of impurities from the walls, which, according to theory, only slightly increases the time of development of hydrodynamic instabilities [2]. In the published experiments the current flowed for only a brief time ($t < 10^{-5}$ sec) and, hence, no adequate information about the stability of such a discharge was obtained.

We concluded from preliminary experiments that a plasma column with a cross section close to that of the chamber was macroscopically stable. Since the discharge took place in a "dirty" plasma, our attention in subsequent experiments was centered on the production of a preheated pure hydrogen plasma, almost radially homogeneous, with a temperature of more than 1 eV by means of a rapid impulsive discharge. The main heating current with a duration of about 100 μsec was then introduced into the plasma produced by this discharge. The rate of current increase was chosen so that the plasma was contained mainly by the magnetic field of its own current.

The reason for the absence of macroscopic instability of the plasma, despite theoretical predictions [3], must be attributed to properties of a discharge in a dense plasma which are neglected in the simplified magnetohydrodynamic model. We can postulate that in certain conditions unstable modes are present but their amplitude is greatly restricted by nonlinear effects (finite conductivity, dissipative processes, radiation, etc.).

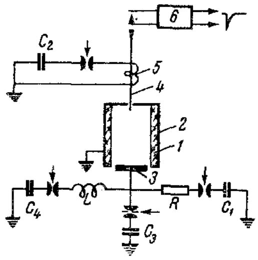

Fig. 1. Diagram of apparatus.

1. A diagram of the experimental apparatus is shown in Fig. 1. The discharge took place in a ceramic chamber 1 with inside diameter 5 cm and height 15 cm. This chamber was fitted into a steel cylinder 2, which was the return current conductor. The experiments were conducted with hydrogen at an initial pressure of 20 mm Hg. Plasma was first obtained by a short impulsive discharge with a current of 100 kA. To ensure breakdown of the gas gap at some distance from the chamber walls we used a low-current discharge (0.5 A) between the potential electrode 3 and the movable part of the grounded electrode 4. The movable electrode was a steel rod 0.2 cm in diameter and in its initial position was at a distance of 0.2 cm from electrode 3. The rod was moved by an electrodynamic "hammer" 5; the time of motion was 7 msec. On emerging from the working chamber the movable electrode in its final position triggered the "ready" circuit 6, which delivered the initiating pulse for the fast impulsive discharge. Current was then delivered (with a delay of up to 10 μsec from the start of the rapid discharge) into the plasma formed by this discharge from a capacitor bank ($C_4 = 1.5 \cdot 10^{-3}$ F, U = 5 kV). The rate of rise of this main current was regulated by the incorporation of an appropriate inductor L and was 5 kA/μsec, while the current amplitude varied up to 200 kA.

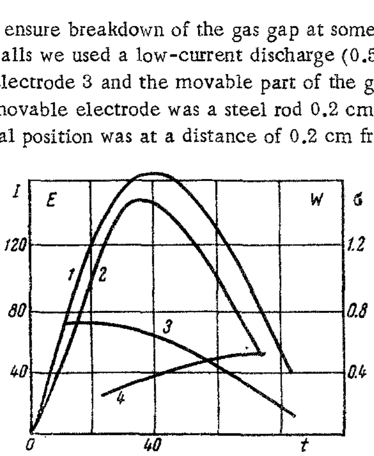

Fig. 2. Electrical characteristics of discharge (t, μsec); p_0 = 20 mm Hg. Hydrogen. 1) Current I, kA; 2) power W, MW/cm^3, delivered to plasma; 3) electric field strength E, V/cm; 4) conductivity σ of plasma, mho/cm.

In the experiment we took high-speed photographs of the discharge, made a time scan of the radiation spectrum, and measured the gas-kinetic pressure, the longitudinal electric field, and the components of the magnetic field.

The behavior of the plasma column was investigated with an SFR-2M streak camera. The ordinary photographs were taken through the quartz window of the transverse slit of the chamber and slow-motion photographs were taken through a series of holes (d = 0.2 cm) in the wall of the steel cylinder. In the latter case the ceramic chamber was replaced by a clear plastic chamber. In both kinds of photography we used a filter which absorbed the red region of the spectrum (more than 5000 Å).

A time scan of the spectrum was obtained by using a rotating disk with a radial slit mounted directly in front of the entrance slit of an ISP-30 quartz spectrograph. The resolving time of the apparatus was 0.5 μsec. The spectrum was recorded from the center of the chamber through the quartz window (d = 1) on aerial film with a sensitivity of 1200 GOST units. The spectral sensitivity of the film was determined by means of hydrogen and mercury-vapor lamps operating under conditions in which the relative spectral distribution of the emitted energy was known.

The magnetic field of the discharge was investigated by means of magnetic probes, insulated from the plasma, and consisting of multiturn coils 0.3 cm in diameter. The probes were located in the walls of the ceramic chamber so that they would not disturb the plasma. Connection of the probes in opposition to one another in a plane perpendicular to the axis of the chamber increased the sensitivity of the measurements and enabled us to follow the development of instabilities and the radial displacement of the current channel to within 0.05 cm.

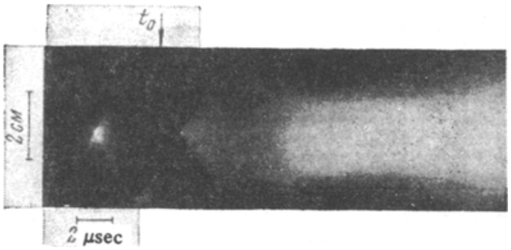

Fig. 3. Streak photograph of discharge.

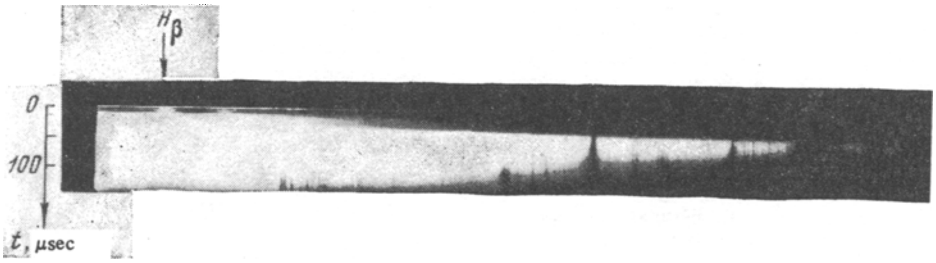

Fig. 4. Time scan of radiation spectrum; H_β is the hydrogen line of Balmer series (4681 A).

The gas-kinetic pressure was measured by piezoelectric transducers near the wall and on the chamber axis close to the electrode. The transducers and the measuring equipment were at the same potential as the plasma. The piezoelectric transducer consisted of a barium titanate cylinder 1 cm in diameter and 0.2 cm high and an acoustically matched brass rod 50 cm long. The design of the transducer ruled out the possibility of mechanical strains leading to distortion of the pressure pulse. The frequency characteristics and sensitivity of the transducers were tested in a shock tube.

The longitudinal electric field was measured in the center of the chamber by two stub or annular electrodes separated from one another by a distance of 1 cm. The discharge current was measured by a calibrated Rogowski coil and the voltage by a capacitance divider.

2. Figure 2 shows curves of the current, electric field, and power W = jE delivered into the plasma. The current practically disappears during the first half-cycle, so that the bulk of the energy (9 kJ) is delivered during the first half-cycle and is 55% of the stored amount.

The streak photograph of the main discharge (Fig. 3) shows the phase of cumulation and divergence of the shock wave of the preliminary discharge, whose region of luminescence is approximately half of the cross section of the chamber; this agreed with the effective cross section of the current channel determined from the pressure measurements. The high-speed photographs of the discharge also confirmed the absence of oscillations. The appearance (after 20 μsec) of a powerful flash of light throughout the chamber made further observation of the current channel impossible. All the experimental data here and henceforth are for hydrogen with an initial pressure p_0 = 20 mm Hg, main current I_0 = 160 kA, current of fast impulsive discharge I_f = 100 kA, and frequency f_f = 500 kHz.

An analysis of probe measurements of the azimuthal and axial components of the magnetic field showed that there were no radial displacements of the plasma filament at any time during the flow of the current, except for small oscillations ($\Delta r \sim 0.2$ cm) at the beginning of the discharge. The streak photographs and probe measurements indicated that there were no macroscopic instabilities of the plasma column.

The optical investigations showed that at the start of the discharge the emission spectrum consisted of hydrogen lines and a continuous spectrum in the visible and ultraviolet regions (6000–2600 Å). Approximately 20 μsec after the start of the current we observed a powerful flash of the continuous spectrum and wall-impurity lines, which subsequently changed to absorption lines (Fig. 4).

Pressure measurements indicated that at the current maximum the gas-kinetic pressure was not completely balanced by the magnetic field and was partially (25%) contained by the walls.

From the results obtained we can construct the following picture of development of the discharge. At the moment when the main current is applied to the chamber there is an almost inhomogeneous, partially ionized hydrogen plasma. Owing to the existing small temperature gradient a current channel with a cross section approximately equal to half that of the chamber is formed at the very start of the discharge. The energy balance in the plasma column is determined by Joule heating and radiation. The radiation energy is expended on heating the cold layers of plasma outside the current channel and on evaporation and heating of the material of the walls. This is indicated by the appearance of impurity lines, an increase in pressure on the walls, and the almost constant cross section of the conducting column of the plasma. The variation of the power delivered to the discharge with time was such that the filament radius was constant throughout the discharge.

3. To verify the main ideas of the mechanism of the heavy-current discharge we consider the very simple model of a discharge in a cylindrical chamber of radius R_0. Using a cylindrical coordinate system we will assume that all the characteristics of the plasma column are independent of z and φ, and depend only on r. This is valid for a long column, where processes at the electrodes can be neglected. This was not quite the case in fact, since the ratio of the chamber length of the column radius was approximately 6.

We use the channel model of a discharge. It is customary to divide the plasma column into a conducting channel 1 of radius R_1 with constant temperature over the radius and a surrounding sheath of cold gas 2. The assumption that the temperature in the conducting channel is constant over the radius is justified by the high thermal conductivity. The current density is constant and the distributions of temperature T and pressure p of the gas are shown in Fig. 5.

An evaluation of the difference in electron and ion temperatures from the energy transfer [4]

$$\frac{T_e - T_i}{T} \approx \frac{j^2 m_i}{e^2 n^2 T} \sim 10^{-2} \tag{3.1}$$

indicates that with sufficient accuracy

$$T_e = T_i = T\,.$$

We derive the energy-balance equation for the sheath. The plasma column is heated by Joule heat and is cooled by radiation and heat conduction. In our conditions the power of the heat loss due to heat conduction is small in comparison with the radiation power and, hence, the heat conduction of the sheath can be neglected. Thus, the pressure in the sheath is determined mainly by absorption of emission of the current channel and the influx of material evaporated from the walls by radiation. For simplicity we will assume that the radiation energy is absorbed in the sheath and is converted to pressure with some coefficient γ. The order of magnitude of this coefficient can be evaluated by assuming, for instance, that the temperature of region (2) is constant. The change in pressure is

$$\Delta p_2 = T\Delta n, \qquad \Delta n = n\,\frac{W_r}{J + p_2}.$$

Here J is the potential energy of evaporation and ionization; p_2 is the pressure in the sheath; W_r is the power of radiation from the current channel. Hence, for the recuperation coefficient we have

$$\gamma = \frac{p_2}{J + p_2} \approx 0.1\,.$$

Hence, the equation of the energy balance per unit length is

$$\frac{3}{2}\frac{dp_2}{dt}(\pi R_0^2 - \pi R_1^2) - \frac{5}{2}\,p_2\frac{d}{dt}(\pi R_0^2 - \pi R_1^2) = \gamma W_r \pi R_1^2\,. \tag{3.2}$$

We derive the energy-balance equation for the current channel. We will assume that the plasma in the channel is completely ionized from the very beginning. The rapid decrease in the conductivity [5] and the total radiation power when the temperature is below $T_0 \approx 1$ eV can be taken into account by approximating the curve of radiation power by the function

$$W_z(T < T_0) = 0, \qquad W_r(T > T_0) = W_r\,.$$

The electron density in the discharge can be about 10^{18} cm^{-3} (on the basis of an initial pressure of 20 mm Hg) and, hence, at temperatures of 1–5 eV emission due to recombination at the ground level may be cut off. Since the temperature over the radius in the channel model is constant, we cannot use the radiative transfer equation. Hence, we take into account the power lost due to radiation by a simpler method—we assume that the plasma radiates as a blackbody in the frequency interval ν', ν'', where $k(\nu)r > 1$. The accuracy of such a procedure is sufficient.

Thus, the radiation power can be written in the form

$$W_r = \varepsilon_* + \varepsilon_r + \frac{2}{R_1}2\pi\int_{\nu'}^{\nu''}\varepsilon(\nu)\,d\nu + 4\pi\int_{\nu''}^{\infty}\varepsilon_L(\nu)\,d\nu. \tag{3.3}$$

Here $\varepsilon(\nu)$ is Planck's function; $\varepsilon_L(\nu)$ is the spectral power density of the Lyman continum; ε_* is the bremsstrahlung power; ε_r is the power of recombination radiation in the case of transitions to levels with a principal quantum number greater than 1.

The energy-balance equation for the current channel

$$\frac{3}{2}\frac{dP}{dt} + \operatorname{div}\left(\frac{3}{2}P\mathbf{V}\right) + P\operatorname{div}\mathbf{V} = \operatorname{div}\mathbf{q}$$

$$\left(\operatorname{div}\mathbf{q} = \frac{j^2}{\sigma} - W_r,\ \sigma = \sigma_0 T^{3/2}\right) \tag{3.4}$$

is integrated over the cross section, assuming that the current density is constant over the radius and the pressure is, accordingly, parabolic.

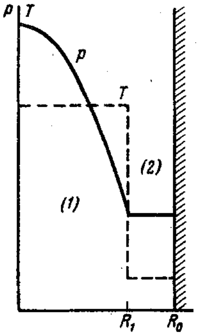

Fig. 5. Pressure and temperature distributions in channel model.

If the discharge time is less than the skin time, then, according to [6], we can assume that

$$V = \dot{\mathbf{r}} = v_0 \mathbf{r}/R_1.$$

After integration Eq. (3.4) has the form

$$\frac{3}{2}\frac{dp_2}{dt}\pi R_1^2 + \left(\frac{5}{2}p_2 + \frac{1}{4}p_1\right)\frac{d(\pi R_1^2)}{dt} + \frac{3}{4}\frac{d}{dt}(p_1 \pi R_1^2) = \frac{I^2}{\sigma \pi R_1^2} - W_r \pi R_1^2 \quad \left(p_1 = \frac{I^2}{\pi R_1^2}\right). \quad (3.5)$$

The plasma temperature in the current channel of radius R_1 is determined from the condition

$$\int_0^{R_1} n(r)\, 2\pi r\, dr = N_0$$

and is

$$T = \frac{p_2 + {}^1\!/_2 p_1}{p_2}\,\frac{\pi R_1^2(t)}{\pi R_1^2(t=0)}\,T_0 . \quad (3.6)$$

In the calculation the experimentally measured total current was used and its form was approximated sufficiently well by a sine curve. Equations (3.2) and (3.5) were solved numerically.

The recuperation coefficient γ, determined from a comparison of the numerical solution with the experimental data, was 0.5. Measurement of the electron concentration by the Stark effect is only possible at the beginning of the discharge and the error involved exceeds the usual 20% due to radial temperature and density gradients.

The experimental and calculated data agree when the density in the current channel is $2 \cdot 10^{18}$ cm^{-3}, which agrees approximately with the electron concentration (10^{18} cm^{-3}) measured from the Stark broadening of H_β at the start of the discharge. Figure 6 shows graphs of the pressure p (atm), the temperature T_0 (eV), and the diameter of the plasma filament D (cm), obtained by calculation. The same figure shows the experimental data. The difference between the experimentally measured and calculated values lies within the limits of experimental error ($\Delta p/p \approx 20\%$, $\Delta T/T \approx 20\%$). In addition, the difference between the calculated and experimental values (especially at the end of the discharge) can be attributed to the simplifications. In particular, γ was assumed to be constant although, in fact, the radiation absorption coefficient and, hence, the recuperation coefficient increases with increase in impurity concentration in the sheath.

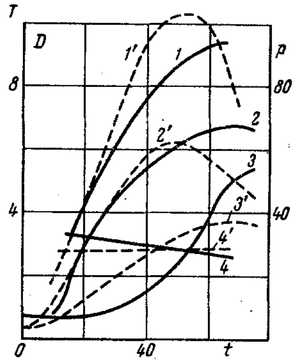

Fig. 6. Comparison of experimental and calculated data. The maximum of the main current corresponds to 37 μsec. The solid curves are the experimental ones and the dashed curves are the calculated ones; 1) and 1') temperature; 2) and 2') pressure on axis; 3) and 3') pressure near wall; 4) and 4') diameter of current channel.

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REFERENCES

1. H. Alfven and E. A. Smars, "Gas-insulation of a hot plasma," Nature, vol. 188, p. 801, 1960.

2. G. S. Murty, "Influence of surrounding neutral medium on the stability of a fluid cylinder carrying an axial current," Arkiv Fysik, vol. 19, p. 511, 1961.

3. B. V. Kadomtsev, "Hydrodynamic instability of a plasma," Questions of Plasma Theory, No. 2 [in Russian], Gosatomizdat, 1963.

4. S. I. Braginskii, "Transport effects in a plasma," Questions of Plasma Theory [in Russian], Gosatomizdat, no. 1, 1963.

5. D. Bresing, "Transport properties of hydrogen," Amer. Inst. Aeronaut. Astron., vol. 13, p. 1422, 1965.

6. S. I. Braginskii and V. D. Shafronov, "A plasma filament in the presence of a longitudinal magnetic field," collection: Plasma Physics and the Problem of Controlled Thermonuclear Reactions, Vol. 2 [in Russian], Izd-vo AN SSSR, 1958.

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